

# INFINITE NUCLEAR MATTER ON THE LIGHT FRONT: A MODERN APPROACH TO BRUECKNER THEORY

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Understanding an important class of experiments requires that light-front dynamics and the related light cone variables  $k^+, k_\perp$  be used. If one uses  $k^+ = k^0 + k^3$  as a momentum variable the corresponding canonical spatial variable is  $x^- = x^0 - x^3$  and the time variable is  $x^0 + x^3$ . This is the light front (LF) approach of Dirac. A relativistic light front formulation of nuclear dynamics is developed and applied to treating infinite nuclear matter in a method which includes the correlations of pairs of nucleons. This is light front Brueckner theory.

## 1 Outline

This talk is divided into four parts. (1) What is the Light Front Approach? The basic idea is to use a “time” variable  $c\tau = ct + z$  (2) Why use it? Certain kinds of high energy experiments are best analyzed using light front or light cone variables. (3) Mean field theory results. (4) Nucleon-nucleon correlations. The way to include these, in any formalism, is Brueckner Theory.

## 2 What is Light Front Dynamics?

This is a relativistic treatment of many-body dynamics<sup>1</sup> in which the “time” variable is taken to be

$$\tau = ct + z = x^0 + x^3 \equiv x^+. \quad (1)$$

The canonically conjugate “energy” variable is  $p^0 - p^3 \equiv p^-$ . One of the “space” variables must be the orthogonal combination  $x^- \equiv t - z$ , with its canonically conjugate momentum:  $p^+ = p^0 + p^3 \equiv p^+$ . The other variables are  $\vec{x}_\perp$ , and  $\vec{p}_\perp$ .

Our notation is  $A^\pm \equiv A^0 \pm A^3$ . The point of this was noticed long ago by experimentalists. Consider a particle with a large velocity such that:  $\vec{v} \approx c\hat{e}_3$ . In that case the momentum  $p^+$  is BIG. An important consequence of using light front variables is that the usual relation between energy and momentum,  $p^\mu p_\mu = m^2$  becomes

$$p^- = \frac{1}{p^+}(p_\perp^2 + m^2), \quad (2)$$

so that one obtains a relativistic kinetic energy without a square root operator. Eq. (2) is of great use in separating relative and center-of-mass variables. Another feature is that here the vacuum is empty. It contains no virtual-pair states.

### 3 Motivation

It is certainly possible to do quantum mechanics this way, but why? I think that the use of light front dynamics is mandated if one wants to correctly understand a large class of high energy nuclear reactions. The most prominent example is deep inelastic lepton scattering from nuclei.

#### 3.1 $x_{Bj}$ , Light Front Nuclear Physics and the EMC effect

Deep inelastic scattering occurs when a quark of four-momentum  $q$  strikes a quark of momentum  $p$  that originated from a nucleon of momentum  $k$ . In that case,

$$x_{Bj} \equiv \frac{-q^2}{2Mq^0} = \frac{p^+}{k^+}, \quad (3)$$

for large enough values of  $-q^2$  and  $q^0$ . One studies, experimentally and theoretically, the ratio of a cross section(per nucleon)  $\sigma(A)$  on a nucleus to that  $\sigma(N)$  on a nucleon. At high energies and momentum transfer one might think that the ratio  $\sigma(A)/\sigma(N)$  would be very close to unity. The European Muon Collaboration found that this was not so—there is a depletion (EMC effect)  $\sigma(A)/\sigma(N) \approx 0.85$  in the region  $x_{Bj} \approx 0.5$  for which valence quark are dominant <sup>2</sup>. If there is such a depletion, and momentum is conserved, there must be an enhancement of the momenta carried by other degrees of freedom. This could be manifest as an enhancement of nuclear pions for  $x_{Bj} \approx 0.1$ . But there were many non-conventional theories of this effect including swollen nucleus, six-quark cluster, and color conductivity through the entire nucleus. Almost immediately after the EMC effect was discovered we argued <sup>3</sup> that another kind of experiment: Drell-Yan production of muon pairs, could be used to test the various theories of the EMC effect. No excess pions were discovered <sup>4</sup> and this was termed a crisis in nuclear physics by Bertsch et al <sup>5</sup>.

My opinion is that the conventional explanation of nuclear binding and related Fermi motion effects has never been properly evaluated because of the failure to re-derive nuclear wave functions using the formalism (as given in reviews <sup>6</sup>) of light front dynamics. Thus, it has been our intention to provide realistic and relativistic calculations of nuclear wave functions using light front dynamics <sup>7–10</sup>.

#### 3.2 Formal aspects

To make light front-nuclear physics calculations we need to know the probability that a nucleon has a given value of  $k^+$ :  $f_N(k^+)$ . Similarly the distribution function for a pion is given by  $f_\pi(k^+)$ . I have emphasized deep inelastic scattering so far, but these quantities enter into the analysis of many experiments including the (e,e'p) and (p,pp) reactions <sup>11</sup>. The consequence of taking  $\tau$  of Eq. (1) as the time variable is that the distribution functions  $f_{N,\pi}$  are simply related to the absolute square of the ground state wave function. If one uses the conventional equal time formulation, one finds that the same information is encoded in the response function which involves matrix elements between the ground and an infinite number of excited states.

In light front dynamics, one only needs the ground state, but one has to obtain this from a consistent calculation. To illustrate the difficulty one may ask, “What is  $k^+$ ?”. Many authors, including myself, have used the idea that  $k^+$  is an energy plus momentum to invoke a relation:  $k^+ = M - \epsilon_\alpha = k^3$ , where  $\epsilon_\alpha$  is an orbital binding energy. This relation is not correct. The variable  $k^+$  is a continuous kinematic variable (akin to  $k^3$  of the usual quantum mechanics). It is not related to any discrete eigenvalue.

#### 4 Light Front Quantization

Our motto is that we need a  $\mathcal{L}$ , no matter how bad! This is necessary in order to derive expressions for the operators  $P^\pm$  which are the “momentum” and “Hamiltonian” of the theory. Consider for example, the Walecka model <sup>12</sup> (also called QHD1)  $\mathcal{L}(\phi, V^\mu, \psi)$ . The degrees of freedom are nucleon  $\psi$ , neutral vector meson  $V^\mu$ , and scalar meson  $\phi$ . This is the simplest Lagrangian that can provide even a caricature of nuclear physics. Exchange of scalar mesons leads to a long ranged attractive potential and exchange of vector mesons leads to a shorter range and stronger repulsive potential. In this way, the nucleons are held together, but are not allowed to collapse. Given  $\mathcal{L}$ , one constructs the energy-momentum tensor,  $T^{\mu\nu}$ . In particular,

$$P^\mu = \frac{1}{2} \int d^2x_\perp dx^- T^{+\mu}. \quad (4)$$

A technical challenge is to express  $T^{+\mu}$  in terms of independent variables. For example, the nucleon is usually treated as a 4-component spinor. But this particle has spin 1/2, so there are really only two independent degrees of freedom, denoted as  $\psi_+$ . One must express the remaining degrees of freedom in terms of  $\psi_+$  <sup>6,7</sup>.

#### 5 Infinite Nuclear Matter in Mean Field Approximation-MFA

This simple limiting case is the first problem we consider. The idea behind the mean field approximation is that the sources of mesons are strong, so there are many mesons, which can be treated as classical fields. The volume is taken as infinite, so that all positions, and spatial-directions are equivalent. We treat nuclear matter in its rest frame here. In that case the solution of the mesonic field equations lead to the results

$$V^\pm = V^0 = \frac{g_v}{m_v^2} \langle \psi^\dagger(0) \psi(0) \rangle; \quad V_i = 0, \quad (5)$$

$$\phi = \frac{-g_s}{m_s^2} \langle \bar{\psi}(0) \psi(0) \rangle, \quad (6)$$

in which the brackets represent ground state matrix elements. The fields  $\phi, V^\pm$  are constants, so the nucleon modes are plane waves. One has a Fermi gas in which  $\psi \sim e^{ik \cdot x}$  and

$$i\partial^- \psi_+ = g_v \bar{V}^- \psi_+ + \frac{k_\perp^2 + (M + g_s \phi)^2}{k^+} \psi_+. \quad (7)$$

The equations (5)–(7) are a self-consistent set of equations.

### 5.1 Nuclear Momentum Content

One uses the energy-momentum tensor to determine  $P^\pm$ . One finds

$$\frac{P^-}{\Omega} = m_s^2 \phi^2 + \frac{4}{(2\pi)^3} \int_F d^2 k_\perp dk^+ \frac{k_\perp^2 + (M + g_s \phi)^2}{k^+}, \quad (8)$$

$$\frac{P^+}{\Omega} = m_v^2 (V^-)^2 + \frac{4}{(2\pi)^3} \int_F d^2 k_\perp dk^+ k^+, \quad (9)$$

in which  $\Omega$  is the volume of the system. The Fermi sphere is determined by using an implicit definition of  $k^3$ :

$$k^+ \equiv \sqrt{(M + g_s \phi)^2 + \vec{k}^2} + k^3. \quad (10)$$

Then one may show that the energy of the nucleus,  $E \equiv \frac{1}{2} (P^- + P^+)$  is the same <sup>7</sup> as for the Walecka model. This is a nice check on the calculation because that model as been worked out in a manifestly covariant manner. Then the minimization,  $\left( \frac{\partial(E/A)}{\partial k_F} \right)_\Omega = 0$  determines the value of the Fermi momentum,  $k_F$ . This very same equation also sets  $P^+ = P^- = M_A$ , a most welcome result.

### 5.2 Mean field results and implications

The numerical calculation shows that the LF reproduces standard good results for energy and density. But the explicit decomposition (9) allows us to determine that nucleons carry only 65% of the nuclear + Momentum ( $M_A$ ). A value of 90% is needed to explain the EMC effect (in infinite nuclear matter) <sup>13</sup>, so this is a problem. Furthermore, vector mesons carry a huge 35% of the + momentum. Because  $V^-$  is constant in space-time,  $V^- \neq 0$  only if  $k^+ \rightarrow 0$ . This means that this effect requires a beam of infinite energy to be detected. These results, which conflict with experiments, might be artifacts of using infinite nuclear matter, or caused by the use of the MFA. More seriously, the  $\mathcal{L}$  could be at fault.

### 5.3 Saving Mean Mean Field Theory?

A simple way to improve the phenomenology is to modify  $\mathcal{L}$ , by for example including scalar meson self coupling terms:  $\phi^3, \phi^4$ . A wide variety of parameter sets reproduce the binding energy and density of nuclear nuclear matter <sup>14</sup>. For one set, nucleons carry 90% of  $P^+$ , so that vector mesons carry 10%. This could be acceptable. of  $P^+$  There is a problem with this parameter set, the related nuclear spin-orbit splitting is found to be too small <sup>15</sup>. This is not so bad, since there are a variety of non-mean field mechanisms which can supply a spin orbit force. Thus one finds a need to go beyond mean field theory. This involves the introduction of light front Brueckner theory.

## 6 Light Front NN interaction

The nucleon-nucleon potential is obtained from one boson exchange using another  $\mathcal{L}$  which includes the effects of pions and other mesons absent from QHD1 and in

which chiral symmetry is respected <sup>10</sup>. The  $\tau$ -ordered perturbation theory rules give expressions <sup>7</sup> which can be translated into the usual language. Schematically, in momentum space we have

$$V(meson) \sim \frac{1}{q_0^2 - \vec{q}^2 - \mu^2} \quad (11)$$

in which  $q^\mu$  is the four-momentum transferred between nucleons and  $\mu$  is the meson mass. This is the standard Yukawa form, except that the effects of retardation are included via the  $q_0$  term. The kernel  $\mathcal{K}$  is the sum of the meson exchanges:

$$\mathcal{K} = \sum_{meson} V(meson), \quad (12)$$

in which the mesons are the usual set of  $\pi, \rho, \omega, \sigma, \eta, \delta$ . The potentials are strong so that their effects are taken into account to all orders by solving the light front version of the Lippman-Schwinger equation <sup>11</sup>. Schematically we write:

$$\mathcal{M} = \mathcal{K} + \int \frac{d^2 p_\perp d\alpha}{\alpha(1-\alpha)} \mathcal{K} \frac{2M^2}{P^2 - \frac{p_\perp^2 + M^2}{\alpha(1-\alpha)} + i\epsilon} \mathcal{M}, \quad (13)$$

in which  $P$  is the total four-momentum of the two nucleon system and  $p_\perp, \alpha$  are relative momenta <sup>7</sup>. This equation does not seem to have rotational invariance, but this can be recovered by making a change of variables in which the z-component of the relative momentum is defined implicitly:

$$\alpha \equiv \frac{E(p) + p^3}{2E(p)}, \quad (14)$$

with  $E(p) = \sqrt{p_\perp^2 + p_3^2 + M^2}$ . Then the integrand in the equation above is simplified:

$$\frac{d^2 p_\perp d\alpha}{\alpha(1-\alpha)} \frac{2M^2}{P^2 - \frac{p_\perp^2 + M^2}{\alpha(1-\alpha)} + i\epsilon} \rightarrow \frac{M^2}{E(p)} \frac{d^3 p}{P^2/4 - E^2(p) + i\epsilon}. \quad (15)$$

This is of the form of the Blankenbecler Sugar equation except that the effects of retardation must be included.

Given this formalism we followed the usual prescription of varying the meson-nucleon form factors to achieve a reasonably good description of the data <sup>10</sup>.

## 7 Light Front Theory of $\infty$ Matter - with NN Correlations

I outline our detailed theory <sup>10</sup>. The starting point is a Lagrangian decomposed into nucleon kinetic terms  $\mathcal{L}_0(N)$ , meson kinetic terms  $\mathcal{L}_0(\text{mesons})$  and meson-nucleon interactions  $\mathcal{L}_I(N, \text{mesons})$ . Then

$$\mathcal{L} = \mathcal{L}_0(N) + \mathcal{L}_I(N, \text{mesons}) + \mathcal{L}_0(\text{mesons}). \quad (16)$$

The two-nucleon one-boson-exchange-potential OBEP,  $\mathcal{V}(NN)$ , does not enter so we add it and subtract it:

$$\mathcal{L} = \mathcal{L}_0(N) - \mathcal{V}(NN) + (\mathcal{L}_I(N, \text{mesons}) + \mathcal{L}_0(\text{mesons}) + \mathcal{V}(NN)) \quad (17)$$

The term in parentheses accounts for mesonic content of Fock space. One does perturbation theory in this operator to learn if one has chosen a nucleon-nucleon potential that is consistent with the chosen Lagrangian. The first term of Eq. (17) represents the standard nuclear many body problem. One handles this by introducing the mean field  $MF$ :

$$\mathcal{L}_0(N) - \mathcal{V}(NN) = \mathcal{L}_0(N) - U_{MF} + (U_{MF} - \mathcal{V}(NN)) \quad (18)$$

We choose  $U_{MF}$  in the usual way, according to the independent pair approximation. In that case the mean field is the folding of scattering matrix with the nuclear density:

$$U_{MF} \sim (\text{Brueckner G - matrix}) \times \rho. \quad (19)$$

The result of all of these manipulations is that one obtains a full wave function which contains both nucleon-nucleon correlations and explicit mesons. This procedure is very similar to the usual many-body theory evaluated with equal time quantization. I stress the differences. The simplicity of the vacuum allows a relativistic theory to be derived using non-relativistic techniques. We are able to obtain light front plus-momentum distributions for nucleons and mesons. The only technical difference is that we include retardation effects in our OBEP.

### 7.1 Saturation Properties

We find good results. The binding energy per nucleon is 14.7 MeV and  $k_F = 1.37$  Fm. The compressibility is 180 MeV. Given this, the interesting thing to do is to assess the influence of this calculation on nuclear structure functions.

## 8 Deep Inelastic Scattering and Drell-Yan Production

We find  $M + g_s\phi = 0.79M$  this is very much larger than the mean field value of  $0.56M$ . As a result nucleons carry more than 84% of the nuclear plus-momentum. The 84% is obtained using only the uncorrelated- Fermi gas part of the wave function. We also estimate that including the 2p-2h correlations would lead to nucleons carrying more than 90% of the plus momentum. Including nucleons with momentum greater than  $k_F$  would substantially increase the computed ratio  $F_{2A}/F_{2N}$  because  $F_{2N}(x)$  decreases very rapidly with increasing values of  $x$  and because  $M^*$  would increase at high momenta. This is a good start to solving the problems mentioned in the earlier parts of this talk. Furthermore, we computed the total number of excess pions, and find that  $\frac{N_\pi}{A} = 5\%$ . This is much smaller than the only previously computed result <sup>16</sup> of 15%. The quantity  $N_\pi$  is not a direct input into computations, but previous phenomenological calculations <sup>17</sup> allow us to hope that the 5% would be consistent with Drell Yan data. Our present conclusion is that light front dynamics leads to reasonable nuclear dynamics. The 90%, and 5% numbers are an excellent start.

Clearly, many things remain to be done with this approach. In the meantime, I would like to emphasize that Light Front Nuclear Physics exists! One can use it to understand any high energy nuclear reaction.

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